problem session to 7.1

Problem Session - Section 7.1

<u>6, p181</u> $\underline{a * a = e}$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ identity is $(0,0)$

operation is the component-wise addition

this group is not <u>cyclic</u>, in particular $\mathbb{Z}_2 \times \mathbb{Z}_2 \not\cong \mathbb{Z}_4$

<u>18 p182</u>

Composition of functions is associative

Identity $i(x) = x$

Inverses $h^{-1} = h$ $j^{-1} = j$ $k^{-1} = k$ $g^{-1} = f$ $f^{-1} = g$

$$h^2(x) = (h \circ h)(x) = h(h(x)) = \frac{1}{1/x} = x$$

$$h^2 = i$$

The set of 6 functions is closed under composition - 36 calculations

$$f^3 = i \quad j^2 = i \quad f^2 j = jf$$

Consider all compositions of powers of f and powers of j:

$$\iota,\ f,\ \underset{\overset{\shortparallel}{g}}{f^2},\ j,\ \underset{\overset{\shortparallel}{h}}{fj},\ \underset{\overset{\shortparallel}{k}}{f^2 j}$$

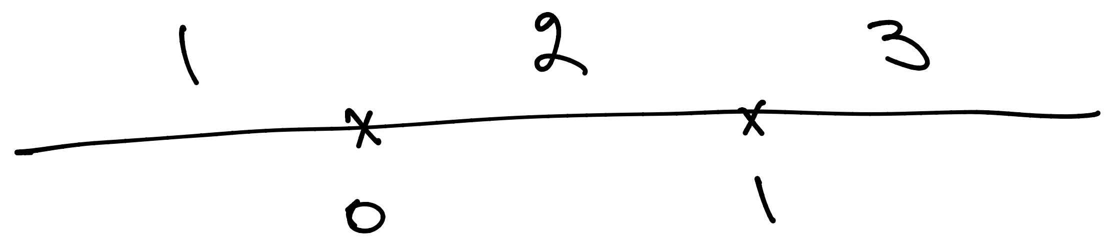
- nothing else can come out.

Thus this set of 6 functions is closed under composition

Let us identify this group out of 6 elements $\Big\}$ p 245 $\mathbb{Z}_6$ or $\underline{\underline{S_3}}$

The group should be isomorphic S_3, and it is interesting to present an isomorphism.

All 6 functions are defined on $K = \{x \in \mathbb{R} \mid x \neq 0, x = 1\}$



Every function (out of the given set of six) performs a permutation of these three intervals.

The composition of function performs a composition of these permutations

$$f \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad j \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$g \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad k \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$h \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ $\quad i \longmapsto \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$j(x) = 1 - x$

31, p183 $\quad A(T)$

For every permutation of the 3 elements,
the group $A(T)$ contains an element which performs
this permutation, and maps every other element of T to itself.
In this way, we have S_3 as a subgroup of $A(T)$.
Since S_3 is not abelian, same is true about $A(T)$.

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35 p183 $\quad f \in S_n \quad$ There exists $k > 0$, integer such $f^k = I$.

a finite group $\quad f, f^2, f^3, \ldots$

$|S_n| = n!$ $\qquad$ there is $m < n$ s.t. $f^n = f^m$

$f^{n-m} = I$

Rem In fact, $f^{n!} = I$ for every $f \in S_n$ (

33, p 183 $\quad T_{a,b} : \mathbb{R} \to \mathbb{R} \qquad T_{a,b}(x) = ax + b$

$G = \{ T_{a,b} \mid a, b \in \mathbb{R},\ a \neq 0 \}$ - non-abelian group under composition of functions

$(\underline{T_{a,b} \circ T_{c,d}})(x) = T_{a,b}(T_{c,d}(x))$

$= T_{a,b}(cx + d) = a(cx + d) + b = acx + ad + b = \underline{T_{ac,\, ad+b}}(x)$

$T_{1,0}$ - the identity $\qquad T_{1,0}(x) = x$

Inverse $\quad T_{a,b} \circ T_{1/a,\, -b/a} = T_{1,0}$

$(a,b) \circ (c,d) = (ac, ad + b)$

25, p 182 $\quad$ Set: $\mathbb{R}^* \times \mathbb{R}$

Operation $\quad (a,b) \# (c,d) = (ac, bc + d)$

$T_{c,d} \circ T_{a,b} = \underline{T_{ac,\, bc+d}}$

31: $(a,b) * (c,d) = (ac, ad + b)$ - is a group

21, p 182 $\quad$ If G is a group with operation $*$, then the same set G with operation $\#$ defined by $a \# b = b * a$ is also a group

That allows you to switch between problems 25 and 33.